

Solución.

Como $[-1, 1] = [-1, 0) \cup [0, 1]$, la hipótesis puede escribirse en 2 casos:

Caso 1. Cuando $x \in [-1, 0)$

$$\begin{aligned} (1) \quad & |x| = -x \\ (2) \quad & E = \left\lfloor \left\lfloor \frac{|x|-2}{3-x} \right\rfloor \right\rfloor = \left\lfloor \left\lfloor \frac{-x-2}{3-x} \right\rfloor \right\rfloor \\ (3) \quad & E = \left\lfloor \left\lfloor 1 + \frac{5}{x-3} \right\rfloor \right\rfloor \\ (4) \quad & -1 \leq x < 0 \\ (5) \quad & -4 \leq x - 3 < -3 \\ (6) \quad & -\frac{1}{3} < \frac{1}{x-3} < -\frac{1}{4} \\ (7) \quad & -\frac{5}{3} < \frac{5}{x-3} < -\frac{5}{4} \\ (8) \quad & -\frac{2}{3} < 1 + \frac{5}{x-3} < -\frac{1}{4} \\ (9) \quad & -1 \leq -\frac{2}{3} < 1 + \frac{5}{x-3} < -\frac{1}{4} < 0 \\ (10) \quad & -1 \leq \frac{-x-2}{3-x} < 0 \\ (11) \quad & \therefore \left\lfloor \left\lfloor \frac{|x|-2}{3-x} \right\rfloor \right\rfloor = -1 \end{aligned}$$

Caso 2. Cuando $x \in [0, 1]$

$$\begin{aligned} (1) \quad & |x| = x \\ (2) \quad & E = \left\lfloor \left\lfloor \frac{|x|-2}{3-x} \right\rfloor \right\rfloor = \left\lfloor \left\lfloor \frac{x-2}{3-x} \right\rfloor \right\rfloor \\ (3) \quad & E = \left\lfloor \left\lfloor -1 + \frac{1}{3-x} \right\rfloor \right\rfloor \\ (4) \quad & 0 \leq x \leq 1 \\ (5) \quad & 2 \leq 3 - x \leq 3 \\ (6) \quad & \frac{1}{3} \leq \frac{1}{3-x} \leq \frac{1}{2} \\ (7) \quad & -\frac{2}{3} \leq \frac{1}{3-x} \leq -\frac{1}{2} \\ (8) \quad & -1 \leq -\frac{2}{3} < -1 + \frac{1}{3-x} \leq -\frac{1}{2} < 0 \\ (9) \quad & -1 \leq \frac{x-2}{3-x} < 0 \\ (10) \quad & -1 \leq \frac{|x|-2}{3-x} < 0 \\ (11) \quad & \therefore \left\lfloor \left\lfloor \frac{|x|-2}{3-x} \right\rfloor \right\rfloor = -1 \end{aligned}$$

Rpta: $E = -1, \forall x \in [-1, 1]$ ■